Massachusetts Institute of Technology
Department of Economics
Working Paper Series

Information Aggregation and Equilibrium Multiplicity
Morris-Shin Meets Grossman-Stiglitz

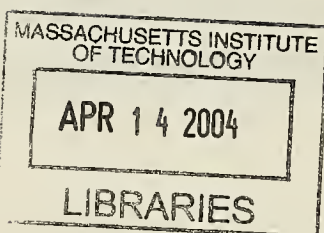George-Marios Angeletos
Ivan Werning

Room E52-251
50 Memorial Drive
Cambridge, MA 02142

Information Aggregation and Equilibrium Multiplicity:
Morris-Shin Meets Grossman-Stiglitz[*]

George-Marios Angeletos
MIT and NBER
angelet@mit.edu

Iván Werning
MIT, NBER and UTDT
iwerning@mit.edu

This draft: February 2004

Abstract

This paper argues that adding endogenous information aggregation to situations where coordination is important – such as riots, self-fulfilling currency crises, bank runs, debt crises or financial crashes – yields novel insights into multiplicity and characterization of equilibria. Morris and Shin (1998) have highlighted the importance of the information structure for this question. They also show that, with exogenous information, multiplicity collapses when individuals observe fundamentals with small enough idiosyncratic noise. In the spirit of Grossman and Stiglitz (1976), we endogenize public information by allowing individuals to observe financial prices or other noisy indicators of aggregate activity. In equilibrium these indicators imperfectly aggregate disperse private information without ever inducing common knowledge. Importantly, their informativeness increases with the precision of private information. We show that multiplicity may survive and characterize the conditions under which it obtains. Interestingly, endogenous information typically reverses the limit result: multiplicity is *ensured* when individuals observe fundamentals with small enough idiosyncratic noise.

JEL Codes: D8, E5, F3, G1.
Keywords: Multiple equilibria, coordination, self-fulfilling expectations, speculative attacks, currency crises, bank runs, financial crashes, rational-expectations, global games.

1 Introduction

It's a love-hate relationship, economists are at once fascinated and uncomfortable with multiple equilibria. On the one hand, a variety of phenomena seem characterized by large

and abrupt changes in outcomes not obviously triggered by commensurate changes in fundamentals. Commentators often attribute these changes to arbitrary changes in 'market sentiments' or 'animal spirits'. Models with multiple equilibria may formally capture these ideas. Prominent examples include self-fulfilling bank runs, currency attacks, debt crises, financial crashes, riots and political regime changes.[1] In this class of models, multiplicity arises due to a coordination problem: attacking a 'regime' – for instance, a currency peg – is beneficial if and only if enough agents are expected to attack.

On the other hand, models with multiple equilibria are sometimes viewed as incomplete theories that should ultimately be extended in some dimension to resolve the indeterminacy. Recently, Morris and Shin (1998)[2] have contributed to this perspective by enriching the information structure away from common knowledge. They show that a unique equilibrium survives when individuals observe fundamentals with small enough idiosyncratic noise. Their analysis is particularly attractive because it can be viewed as a small perturbation around the original common-knowledge model.

More generally, Morris and Shin introduce a useful framework for studying how information heterogeneity affects the determinacy and characterization of equilibria. The dispersion of valuable information about uncertain fundamentals plays a critical role for their uniqueness result. An earlier literature dealt with the transmission and aggregation of disperse information in rational-expectations equilibria. In particular, Green (1973) and Grossman (1981) highlight that prices may be excellent aggregators of dispersed information by showing that in some cases they can be fully revealing, yielding common knowledge of economic fundamentals.

Morris-Shin abstracts from the role of financial prices and other indicators as endogenous aggregators of information. Taking the information structure as exogenous is a useful first step and helps isolate the critical role played by disperse beliefs. However, if financial prices convey information about the underlying fundamentals, then the dispersion of beliefs is determined endogenously in equilibrium. Information aggregation can thus play an important role in determining whether multiple equilibria arise. Indeed, Atkeson (2000) notes that multiple equilibria may survive in the extreme case that financial prices are fully revealing and restore common knowledge. Moreover, for most of the applications of interest, it seems unnatural to rule out endogenous information aggregators, such as financial prices or other indicators of economic activity.

In this paper, we make the first attempt, to the best of our knowledge, to incorporate

---

[1] For example, Diamond and Dybvig (1983), Obstfeld (1986, 1996), Velasco (1996), Calvo (1988), Cooper and John (1988), Cole and Kehoe (1996).

[2] Morris and Shin (1998) build on Carlson and van Damme (1993) who developed the Global Games approach for the two-player two-action games. See also Morris and Shin (1998, 2000, 2001, 2003, 2004).

endogenous information aggregation in coordination economies with heterogenous information. Our model takes as given the initially dispersed private information that is crucial in the Morris-Shin framework. We build on this by allowing individuals to observe an economic indicator that, in equilibrium, aggregates disperse information. Importantly, we avoid restoring common knowledge by allowing enough 'noise' in the aggregation process, as in Grossman and Stiglitz (1976, 1980). Thus, none of our results are driven by restoring common knowledge, the main theme in Atkeson's comments.

We consider a variety of endogenous information structures, including indicators other than financial prices. Indeed, we begin our exploration with a model that allows agents to condition their behavior on a noisy signal of the aggregate actions of other agents. We begin with this model for two reasons. First, this situation seems directly relevant in many applications. For instance, during riots or bank runs, an important part of the story is that people are actively watching what others are doing. Ignoring this aspect may be missing an important piece of the puzzle. Second, this model parsimoniously highlights aspects of the problem that recur when financial prices are the instruments for aggregating information.

We study two versions of the observable actions case. In the first version, agents are assumed to observe a noisy signal of contemporaneous aggregate actions. An equilibrium requires that agents choose optimally given the observed signal and, at the same time, this signal be generated by the aggregation of individual choices. Thus, our equilibrium concept is novel and unavoidably at the crossroads of rational-expectations and game theory. The second version avoids the simultaneity in the signal and actions by dividing the population into two groups, 'early' and 'late' agents. Early agents move first and base their decisions to attack solely on their private information. Late agents move second and can observe a noisy signal about the aggregate activity of early agents. This non-simultaneous version only requires standard game-theoretic equilibrium concepts. We show that the equilibria of this second version converge to that of the first as the size of the early movers vanishes.

We next study environments where individuals cannot observe others actions directly but instead can trade in a financial asset market. This market opens after they have received their private information but prior to choosing whether to attack the regime. The rational-expectations equilibrium in the asset market generates imperfect public information that agents use in addition to their private information when they decide whether to attack. This framework opens up new modeling choices regarding the specification of the asset's payoff and the preferences over its risky return. Indeed, we consider four different specifications that can be solved in closed-form.

A common insight emerges from all the cases we study: the precision of endogenous public information is increasing in the exogenous precision of private information. We show that this

implies that introducing endogenous sources of information is important for understanding the likelihood of uniqueness vs. multiplicity of equilibria. Interestingly, for all but one of the six specifications we study, endogenous information reverses the Morris-Shin limiting result: multiplicity is *ensured* when individuals observe fundamentals with small enough idiosyncratic noise. Conversely, uniqueness is ensured if idiosyncratic noise is large enough.

Despite the difference in the limiting result we view our paper as underscoring the general theme emphasized by Morris-Shin, that multiplicity or uniqueness may depend on details of the information structure and that these are worth exploring. This paper has explored the importance of endogenous information aggregation.[3]

The rest of the paper is organized as follows. Section 2 introduces the basic model and reviews the Morris-Shin benchmark with exogenous public information. Section 3 introduces endogenous public information with a signal on aggregate actions. Section 4 studies the role of financial prices as endogenous aggregators of information. Section 5 concludes.

2   The Basic Model

We present an abstract general formulation of the basic model and then briefly discuss the various interpretations available in the literature.

Actions, Outcomes and Payoffs. There are two possible regimes, the status quo and an alternative. There is a measure-one continuum of agents, indexed by $i \in [0, 1]$. Each agent can choose between an action that is favorable to the alternative regime and an action is favorable to the status quo. We call these actions, respectively, "attack" and "not attack". All agents move simultaneously.

We denote the regime outcome with $R \in \{0, 1\}$, where $R = 0$ represents survival of the status quo and $R = 1$ represents collapse. We similarly denote the action of an agent with $a_i \in \{0, 1\}$, where $a_i = 0$ represents "not attack" and $a_i = 1$ represents "attack".

The payoff from not attacking is normalized to zero. The payoff from attacking is $b > 0$ if the status quo is abandoned and $-c < 0$ otherwise. Hence, the utility of agent i is

$$U(a_i, R) = a_i(bR - c).$$

---

[3]Endogenizing the information structure is also the theme in Angeletos, Hellwig and Pavan (2003, 2004). They examine how the information conveyed by active policy interventions may result to multiplicity, and how the evolution of information over time affects the dynamics of regime change. Dasgupta (2002), on the other hand, considers how social learning affects coordination.

4

Finally, the status quo is abandoned ($R = 1$) if and only if

$$A \geq \theta,$$

where $A \equiv \int a_i di \in [0, 1]$ denotes the mass of agents attacking and $\theta \in \mathbb{R}$ parameterizes the exogenous strength of the status quo (or the quality of the economic fundamentals). Let $\underline{\theta} \equiv 0$ and $\overline{\theta} \equiv 1$.

Note that the actions of the agents are strategic complements, since it pays for an individual to attack if and only if the status quo collapses and, in turn, the status quo collapses if and only if a sufficiently large fraction of the agents attacks. This coordination problem is the heart of the model.

Interpretations. This simple model can capture the role of coordination and multiplicity of equilibria in a variety of interesting applications. For instance, in models of self-fulfilling currency crises (Obstfeld, 1986, 1996; Morris and Shin, 1998), there is a central bank interested in maintaining a currency peg and a large number of speculators, with finite wealth, deciding whether to attack the currency or not. In this context, a "regime change" occurs when a sufficiently large mass of speculators attacks the currency, forcing the central bank to abandon the peg.

In models of self-fulling bank runs, a "regime change" occurs once a sufficiently large number of depositors decide to withdraw their deposits, relative to liquid resources available to the system, forcing the bank to suspend its payments. Similarly, in models of self-fulfilling debt crises (Calvo, 1988; Cole and Kehoe, 1996; Morris and Shin; 2003), a "regime change" occurs when a lender fails to obtain refinancing by a sufficiently large fraction of its creditors.

Finally, Atkeson (2000) interprets the model as describing riots. The potential rioters may or may not overwhelm the police force in charge of containing social unrest depending on the number of the rioters and the strength of the police force.

Information. Suppose for a moment that θ were commonly known by all agents. For $\theta \leq \underline{\theta}$, the fundamentals are so weak that the regime is doomed with certainty and the unique equilibrium is every agent attacking. For $\theta > \overline{\theta}$, the fundamentals are so strong that the regime can survive an attack of any size and the unique equilibrium is every agent not attacking.

For intermediate values, $\theta \in (\underline{\theta}, \overline{\theta}]$, the regime is sound but vulnerable to a sufficiently large attack and there are multiple equilibria sustained by self-fulfilling expectations. In one equilibrium, individuals expect everyone else to attack, they then find it individually optimal to attack, the status quo is abandoned and expectations are vindicated. In another, individuals expect no one else to attack, they then find it individually optimal not to attack, the

5

status quo is spared and expectations are again fulfilled. The interval $(\underline{\theta}, \overline{\theta}]$ thus represents the set of "critical fundamentals" for which multiple equilibria are possible under common knowledge.

Implicitly, each equilibrium is sustained by different self-fulfilling expectations about what other agents do. With common knowledge, in equilibrium individuals can perfectly forecast each other actions and coordinate on multiple courses of action. Following Morris and Shin (1998), we assume that θ is never common knowledge and that individuals instead have private noisy information about θ. Private information serves as an anchor for individual's actions that may avoid the indeterminacy of expectations about others actions.

Initially agents have a common prior about θ; for simplicity, we let this prior be (degenerate) uniform over the entire real line. Agent i then observes a private signal

$$x_i = \theta + \xi_i,$$

where the idiosyncratic noise ξ_i is $\mathcal{N}(0, \sigma_x^2)$ with $\sigma_x > 0$ and is independent of θ. The signal x_i is thus a sufficient statistic for the private information of an agent.

Note that because there is a continuum of agents the information contained by the entire economy, $(x_i)_{i \in [0,1]}$, is enough to infer the fundamental θ. However, this information is dispersed throughout the population, which is the key feature of the Morris-Shin framework.

Finally, agents may also have access to some public information. We start by reviewing the Morris-Shin benchmark, where the public signal is *exogenous*. We then consider the *endogenous* public information generated by a noisy indicator of aggregate activity or prices

2.1 The Morris-Shin Benchmark: Exogenous Information

In this subsection, we assume an *exogenous* public signal $z = \theta + v$, where $v \sim \mathcal{N}(0, \sigma_z^2)$ in addition to the private signals $x_i = \theta + \xi_i$, where $\xi_i \sim \mathcal{N}(0, \sigma_x^2)$. The private noise ξ and the public noise v are distributed independently of each other and independently of θ. Our model then reduces to that of Morris and Shin (2000, 2001), with exogenous private and public information (see also Hellwig, 2002).

In a monotone equilibrium, for any realization of z, there is a threshold $x^*(z)$ such that an agent attacks if and only if $x \leq x^*(z)$. By implication, the aggregate size of the attack is decreasing in θ, so that there is also a threshold $\theta^*(z)$ such that the status quo is abandoned if and only if $\theta \leq \theta^*(z)$. A monotone equilibrium is identified by x^* and θ^*. In step 1, below, we characterize the equilibrium θ^* for given x^*. In step 2, we characterize the equilibrium x^* for given θ^*. In step 3, we characterize both conditions and examine equilibrium existence and uniqueness.

Step 1. For given realizations of θ and z, the aggregate size of the attack is given by the mass of agents who receive signals $x \leq x^*(z)$. That is,

$$A(\theta, z) = \Phi\left(\sqrt{\alpha_x}(x^*(z) - \theta)\right),$$

where $\alpha_x = \sigma_x^{-2}$ is the precision of private information. Note that $A(\theta, z)$ is decreasing in θ, so that regime change occurs if and only if $\theta \leq \theta^*(z)$, where $\theta^*(z)$ is the unique solution to

$$A(\theta^*(z), z) = \theta^*(z).$$

Rearranging we obtain:

$$x^*(z) = \theta^*(z) + \frac{1}{\sqrt{\alpha_x}}\Phi^{-1}(\theta^*(z)). \tag{1}$$

Step 2. Given that regime change occurs if and only if $\theta \leq \theta^*(z)$, the payoff of an agent is

$$\mathbb{E}\left[U(a, R(\theta, \varepsilon))|x, z\right] = a(b\Pr\left[\theta \leq \theta^*(z) \mid x, z\right] - c).$$

Let $\alpha_x = \sigma_x^{-2}$ and $\alpha_z = \sigma_z^{-2}$ denote, respectively, the precision of private and public information. The posterior of the agent is

$$\theta \mid x, z \sim \mathcal{N}\left(\delta x + (1-\delta)z, \ \alpha^{-1}\right),$$

where $\delta \equiv \alpha_x/(\alpha_x + \alpha_z)$ is the relative precision of private information and $\alpha \equiv \alpha_x + \alpha_z$ is the overall precision of information. Hence, the posterior probability of regime change is

$$\Pr\left[\theta \leq \theta^*(z) \mid x, z\right] = 1 - \Phi\left(\sqrt{\alpha}(\delta x + (1-\delta)z - \theta^*(z))\right),$$

which is monotonic in x. It follows that the agent attacks if and only if $x \leq x^*(z)$, where $x^*(z)$ solves the indifference condition

$$b\Pr\left[\theta \leq \theta^*(z) \mid x^*(z), z\right] = c.$$

Substituting the expression for the posterior and the definition of δ and α, we obtain:

$$\Phi\left(\sqrt{\alpha_x + \alpha_z}\left(\frac{\alpha_x}{\alpha_x + \alpha_z}x^*(z) + \frac{\alpha_z}{\alpha_x + \alpha_z}z - \theta^*(z)\right)\right) = \frac{b-c}{b}. \tag{2}$$

Step 3. Combining (1) and (2), we conclude that $\theta^*(z)$ can be sustained in equilibrium if and only if it solves

$$G\left(\theta^*(z), z\right) = g, \tag{3}$$

where $g = \sqrt{1 + \alpha_z/\alpha_x} \Phi^{-1}(1 - c/b)$ and

$$G(\theta, z) \equiv \frac{\alpha_z}{\sqrt{\alpha_x}}(z - \theta) + \Phi^{-1}(\theta).$$

With $\theta^*(z)$ given by (3), $x^*(z)$ is then given by (1). We are now in a position to establish existence and determinacy of the equilibrium by considering the properties of the function G. Note that, for every $z \in \mathbb{R}$, $G(\theta, z)$ is continuous in θ, with $G(\underline{\theta}, z) = -\infty$ and $G(\overline{\theta}, z) = \infty$, which implies that there necessarily exists a solution and any solution satisfies $\theta^*(z) \in (\underline{\theta}, \overline{\theta})$. This establishes existence; we now turn to uniqueness. Note that

$$\frac{\partial G(\theta, z)}{\partial \theta} = \frac{1}{\phi(\Phi^{-1}(\theta))} - \frac{\alpha_z}{\sqrt{\alpha_x}}$$

Since $\max_{w \in \mathbb{R}} \phi(w) = 1/\sqrt{2\pi}$ then if $\alpha_z/\sqrt{\alpha_x} \leq \sqrt{2\pi}$ we have that G is strictly increasing in θ, which implies a unique solution to (3). If instead $\alpha_z/\sqrt{\alpha_x} > \sqrt{2\pi}$, then G is non-monotonic in θ and there is an interval $(\underline{z}, \overline{z})$ such that (1) admits multiple solutions $\theta^*(z)$ whenever $z \in (\underline{z}, \overline{z})$ and a unique solution otherwise. We conclude that monotone equilibrium is unique if and only if $\alpha_z/\sqrt{\alpha_x} \leq \sqrt{2\pi}$.

We summarize these results in the following proposition.

Proposition 1 (Morris-Shin) *Suppose agents observe an exogenous public and private signal. Let σ_x and σ_z denote the standard deviations of the private and the public noise, respectively. Monotone equilibria exist and are unique if and only if $\sigma_x/\sigma_z^2 \leq \sqrt{2\pi}$.*

Finally, consider the limits as $\sigma_x \to 0$ for given σ_z, or $\sigma_z \to \infty$ for given σ_x. In either case, $\alpha_z/\sqrt{\alpha_x} \to 0$ and $\sqrt{(\alpha_x + \alpha_z)/\alpha_x} \to 1$. Condition (3) then implies that $\theta^*(z) \to \widehat{\theta} = 1 - c/b$, for all z. This proves the following result, which we refer to as the *Morris-Shin limit result*:

Proposition 2 (Morris-Shin limit) *In the limit as either $\sigma_x \to 0$ for given σ_z, or $\sigma_z \to \infty$ for given σ_x, there is a unique monotone equilibrium in which the regime changes if and only if $\theta \leq \widehat{\theta}$, where $\widehat{\theta} = 1 - c/b \in (\underline{\theta}, \overline{\theta})$.*

3 Endogenous Information on Actions

We now study the case where public information is *endogenous*. Agents no longer receive the public signal z as assumed in section 2.1. Instead, individuals are able to observe a public noisy signal of the aggregate actions of others.

We study two versions of such a model. In the first version, contemporaneous actions are observed with noise. Thus, our equilibrium concept is novel and unavoidably at the crossroads of rational-expectations and game theory.

The second version, in Section 3.3, has non-simultaneous moves by dividing the population into two groups, 'early' and 'late' movers. Individuals in the early group make their decisions to attack or not based solely on their private information. Individuals in the late group move and are able to observe a noisy signal of the early group's aggregate action. This non-simultaneous version only requires standard game-theoretic equilibrium concepts. We show that the equilibria of this second version converge to that of the first as the size of the early movers vanishes.

3.1 Equilibrium with Endogenous Information

We assume that agents can condition their behavior on a noisy indicator of the *contemporaneous* aggregate attack:

$$y = s(A, \varepsilon)$$

where ε is random noise and $s : [0,1] \times \mathbb{R} \to \mathbb{R}$. All agents are assumed to move simultaneously so that y is a signal about contemporaneous actions.

For reasons of tractability we specify the signal function as $s(A, \varepsilon) = \Phi^{-1}(A) + \varepsilon$ and the noise ε as $\mathcal{N}(0, \sigma_\varepsilon^2)$ with $\sigma_\varepsilon > 0$. The common noise ε is distributed independently of the fundamentals θ and the idiosyncratic noise ξ. As we will see, the above specification allows the equilibrium to preserves normality of the information structure, which in turn permits closed-form solutions. This convenient specification was introduced by Dasgupta (2001) in a different setup.

The information structure is parameterized by the pair of standard deviations $(\sigma_x, \sigma_\varepsilon)$. In any symmetric equilibrium agents are distinguished solely by their information, summarized by their observation of the private signal x_i and the public signal y. Let $a(x_i, y)$ denote the action chosen by such an agent. A symmetric rational-expectations equilibrium is defined as follows.

A *symmetric equilibrium* consists of an endogenous signal $y = Y(\theta, \varepsilon)$, an individual

9

attack strategy $a(x, y)$, and an aggregate attack $A(\theta, y)$, that satisfy:

$$a(x, y) \; = \; \arg \max_{a \in [0,1]} \mathbb{E}\left[U(a, R(\theta, y)) \mid x, \; y \right] \tag{4}$$

$$A(\theta, y) \; = \; \int_x a(x, y) d\Phi \left(\frac{x - \theta}{\sigma_x} \right) \tag{5}$$

$$y \; = \; \Phi^{-1}(A(\theta, y)) + \varepsilon \tag{6}$$

for all $(\theta, \varepsilon, x, y) \in \mathbb{R}^4$. Where $R(\theta, y) = 1$ if $A(\theta, y) \geq \theta$ and $R(\theta, y) = 0$ otherwise.

Condition (4) means that $a(x, y)$ is the optimal strategy for the agent given that regime change occurs if and only if $A(\theta, y) \geq \theta$, whereas condition (5) means that $A(\theta, y)$ is simply the aggregate across agents. Of course, the aggregate public signal y must be consistent with individual actions which gives condition (6). This is the rational-expectations feature in our equilibrium concept.

For tractability we focus on symmetric equilibria where the information structure is normally distributed and the strategy of the agents is monotone in private information. As we shall see below, normality of the information structure is an implication of the non-simultaneous model of section 3.3. We refer to such equilibria simply as *monotone equilibria*.

3.2 Equilibrium Analysis

We now study the equilibrium conditions (4)-(6). In monotone equilibria, for any realization of y, there exist thresholds $x^*(y)$ and $\theta^*(y)$ such that an agent attacks if and only if $x \leq x^*(y)$ and the regime changes if and only if $\theta \leq \theta^*(y)$. A monotone equilibrium is thus identified with the triplet of mappings x^*, θ^* and Y.

We construct the set of monotone equilibria in four steps. In Step 1, we start with an arbitrary x^* used by the agents and use conditions (5) and (6) to characterize the implied aggregate attack A, the resulting θ^* and the possible public signals Y. In Step 2, we take θ^* and Y as given and use condition (4) to compute the threshold x^{**} that is individually optimal. In Step 3, we study the fixed point $x^* = x^{**}$. Finally, in Step 4, we consider the determinacy of Y.

Step 1. In a monotone equilibrium, $a(x, y) = 1$ if and only if $x \leq x^*(y)$, for some function x^*. The aggregate attack is then

$$A(\theta, y) = \Phi \left(\sqrt{\alpha_x}(x^*(y) - \theta) \right), \tag{7}$$

10

where $\alpha_x = \sigma_x^{-2}$. Note that $A(\theta, y)$ is decreasing in θ so there exists a function $\theta^*(y)$ such that $A(\theta, y) \geq \theta$ if and only if $\theta \leq \theta^*(y)$. The threshold $\theta^*(y)$ solves $A(\theta^*(y), y) = \theta^*(y)$, or equivalently

$$x^*(y) = \theta^*(y) + \frac{1}{\sqrt{\alpha_x}} \Phi^{-1}(\theta^*(y)). \tag{8}$$

Equilibrium condition (6) implies that the signal signal must satisfy,

$$y = \sqrt{\alpha_x} \left[x^*(y) - \theta \right] + \varepsilon,$$

or equivalently

$$x^*(y) - \sigma_x y = \theta - \sigma_x \varepsilon. \tag{9}$$

For any $(\theta, \varepsilon) \in \mathbb{R}^2$, let $z = \tilde{Z}(\theta, \varepsilon) \equiv \theta - \sigma_x \varepsilon$ and note that (9) is a relation between y and z. De□ne the correspondence

$$\mathcal{Y}(z) = \left\{ y \in \mathbb{R} \mid x^*(y) - \sigma_x y = z \right\}. \tag{10}$$

In Step 4, we show that $\mathcal{Y}(z)$ is non-empty and examine when it is single- or multi-valued. Now take any function $\tilde{Y}(z)$ that is a selection from this correspondence, i.e., such that $\tilde{Y}(z) \in \mathcal{Y}(z)$ for all z, and let the signal be $Y(\theta, \varepsilon) = \tilde{Y}(\tilde{Z}(\theta, \varepsilon)) = \tilde{Y}(\theta - \sigma_x \varepsilon)$. As we shall see any such selection preserves normality of the information structure.

Step 2. We now map θ^* and Y to x^{**}. Given that regime change occurs if and only if $\theta \leq \theta^*(y)$, the expected payoff for the agent is given by

$$\mathbb{E}\left[U(a, R(\theta, \varepsilon)) \mid x, y \right] = a \left\{ b \Pr\left[\theta \leq \theta^*(y) \mid x, y \right] - c \right\}.$$

We thus need to consider the determination of the posterior probability $\Pr\left[\theta \leq \theta^*(y) \mid x, y \right]$. The observation of $y = Y(\theta, \varepsilon) = \tilde{Y}(z)$ is equivalent to the observation of

$$Z(y) \equiv x^*(y) - \sigma_x y = \theta - \sigma_x \varepsilon = z$$

That is, it is as if the agents observe a public signal z about θ, with noise $\mathcal{N}(0, \sigma_x^2 \sigma_\varepsilon^2)$. Recall that each agent also observes a private signal about θ, with idiosyncratic noise $\mathcal{N}(0, \sigma_x^2)$. Let $\alpha_x \equiv \sigma_x^{-2}$, $\alpha_\varepsilon \equiv \sigma_\varepsilon^{-2}$, and $\alpha_z \equiv \alpha_x \alpha_\varepsilon \equiv (\sigma_x \sigma_\varepsilon)^{-2}$. Combining the two sources of information we conclude that the posterior of an agent is

$$\theta \mid x, y \sim \mathcal{N}\left(\delta x + (1 - \delta) Z(y), \alpha^{-1} \right),$$

where $\alpha = \alpha_x + \alpha_z$ is the total precision of information and $\delta = \alpha_x / (\alpha_x + \alpha_z)$ is the precision

11

of x relative to the total.

It follows that

$$\Pr\left[\theta \le \theta^*(y) \mid x, y\right] = 1 - \Phi\left(\sqrt{\alpha}\left(\delta x + (1-\delta)Z(y) - \theta^*(y)\right)\right).$$

Note that the above is monotonic in x so the agent attacks if and only if $x \le x^{**}(y)$, where $x^{**}(y)$ solves

$$b\Pr\left[\,\theta \le \theta^*(y) \mid x^{**}(y), y\,\right] = c.$$

Combining the above two conditions and substituting $Z(y) = x^*(y) - y/\sqrt{\alpha_x}$, we find that $x^{**}(y)$ must solve

$$\Phi\left(\sqrt{\alpha}\left(\delta x^{**}(y) + (1-\delta)\left(x^*(y) - \frac{1}{\sqrt{\alpha_x}}y\right) - \theta^*(y)\right)\right) = \frac{b-c}{b}. \tag{11}$$

Step 3. In equilibrium, $x^{**} = x^*$ and (11) reduces to

$$\Phi\left(\sqrt{\alpha}\left(x^*(y) - \theta^*(y) - \frac{1-\delta}{\sqrt{\alpha_x}}y\right)\right) = \frac{b-c}{b}.$$

Combining the above with (8), using $\delta = \alpha_x/(\alpha_x + \alpha_z)$ and $\alpha = \alpha_x + \alpha_z$, and rearranging, we obtain:

$$\theta^*(y) = \Phi\left(\frac{\alpha_z}{\alpha_x + \alpha_z}y + \sqrt{\frac{\alpha_x}{\alpha_x + \alpha_z}}\Phi^{-1}\left(\frac{b-c}{b}\right)\right), \tag{12}$$

$$x^*(y) = \theta^*(y) + \frac{1}{\sqrt{\alpha_x}}\Phi^{-1}\left(\theta^*(y)\right), \tag{13}$$

where $\alpha_z = \alpha_x\alpha_\varepsilon$. Hence, for all σ_x and σ_ε, the equilibrium x^* and θ^* are determined uniquely and irrespectively of the selected equilibrium signal Y. Moreover, both $\theta^*(y)$ and $x^*(y)$ are increasing in y. Finally, $\theta^*(y)$ does not depend on σ_x, $\theta^*(y) \to 1 - c/b$ as $\sigma_\varepsilon \to \infty$ and $\theta^*(y) \to \Phi(y)$ as $\sigma_\varepsilon \to 0$.

Step 4. We finally need to consider the equilibrium correspondence $\mathcal{Y}(z)$. Recall that this is given by the set of solutions to

$$x^*(y) - \frac{1}{\sqrt{\alpha_x}}y = z.$$

Using (12) and (13), the above reduces to

$$F(y) \equiv \Phi\left(\frac{\alpha_z}{\alpha_x + \alpha_z}y + \Lambda\right) + \frac{1}{\sqrt{\alpha_x}}\left(-\frac{\alpha_x}{\alpha_x + \alpha_z}y + \Lambda\right) = z, \tag{14}$$

where $\Lambda \equiv \sqrt{\alpha_x/(\alpha_x + \alpha_z)}\,\Phi^{-1}(1 - c/b)$. Note that $F(y)$ is continuous in y, and $F(y) \to -\infty$ as $y \to +\infty$, and $F(y) \to +\infty$ as $y \to -\infty$. Thus, the correspondence $\mathcal{Y}(z)$ is non-empty and an equilibrium always exist. To examine multiplicity, we ask whether $\mathcal{Y}(z)$ is single-valued for all z, which is true if and only if F is monotonic in y.

Differentiating F we obtain

$$F'(y) = -\frac{\sqrt{\alpha_x}}{\alpha_x + \alpha_z}\left(1 - \frac{\alpha_z}{\sqrt{\alpha_x}}\phi\left(\frac{\alpha_z}{\alpha_x + \alpha_z}y + \Lambda\right)\right).$$

It follows that the determinacy of equilibrium hinges on the ratio $\alpha_z/\sqrt{\alpha_x}$, like in the Morris-Shin benchmark. Since $\max_{w \in \mathbb{R}} \phi(w) = 1/\sqrt{2\pi}$, we have that F is decreasing in y, and therefore $\mathcal{Y}(z)$ is single-valued for all z, if and only if $\alpha_z/\sqrt{\alpha_x} \leq \sqrt{2\pi}$. If instead $\alpha_z/\sqrt{\alpha_x} > \sqrt{2\pi}$, there are thresholds $\underline{z}$ and $\overline{z}$ such that $\mathcal{Y}(z)$ takes one value for $z \notin (\underline{z}, \overline{z})$ but three values for $z \in (\underline{z}, \overline{z})$. These thresholds are given by $\underline{z} = F(\underline{y})$ and $\overline{z} = F(\overline{y})$, where $\underline{y}$ and $\overline{y}$ are, respectively, the lowest and highest solution with $F'(y) = 0$.

Unlike the Morris-Shin benchmark, however, the ratio $\alpha_z/\sqrt{\alpha_x}$ is endogenous. Using $\alpha_z = \alpha_\varepsilon \alpha_x$, we conclude that the equilibrium is unique if and only if $\alpha_\varepsilon \sqrt{\alpha_x} \leq \sqrt{2\pi}$.

The following proposition summarizes these results.

Proposition 3 (Morris-Shin meet Grossman-Stiglitz) *A monotone equilibrium is characterized by a triplet of mappings (Y, x^*, θ^*) such that the endogenous signal satisfies $y = Y(\theta, \varepsilon)$, a agent attacks if and only if $x \leq x^*(y)$, and regime change occurs if and only if $\theta \leq \theta^*(y)$.*

A monotone equilibrium exists for all $(\sigma_x, \sigma_\varepsilon)$ and is unique if and only if $\sigma_\varepsilon^2 \sigma_x \geq 1/\sqrt{2\pi}$. If $\sigma_\varepsilon^2 \sigma_x < 1/\sqrt{2\pi}$, the equilibrium signal function Y is indeterminate, but the equilibrium threshold functions x^* and θ^* remain unique and independent of the selected Y.

We conclude that the equilibrium is unique only if there is enough noise in both sources of information, the exogenous information of the agents and the endogenous signal about aggregate activity. Multiple equilibria survive as long as either source of information is sufficiently precise.

Interestingly, when multiplicity arises it is with respect to aggregate outcomes but not with respect to individual behavior. To understand this result, consider the common-knowledge limit $(\sigma_x = \sigma_\varepsilon = 0)$, in which case $x = \theta$ and $y = \Phi^{-1}(A)$, so that the agent learns

13

θ perfectly by observing x and learns A by observing y. The agent then finds it optimal to attack if and only if $A \geq \theta$, or equivalently $x \leq \Phi(y)$. Here, the equilibrium strategy $a(x, y)$ for the agent is uniquely determined with $x^*(y) = \Phi(y)$. However, the equilibrium values of A and y are not uniquely determined. Instead, for every $\theta \in (\underline{\theta}, \overline{\theta}]$, both $A = 0$ and $A = 1$ can be sustained in equilibrium.[4] When σ_x and σ_ε are non-zero, the same nature of indeterminacy remains. The equilibrium behavior of the agent is uniquely determined for any given observation x and y, but there can be multiple equilibrium values of A and y for any given realization of θ and ε.

Finally, since our endogenous-information economy is different from the exogenous information economy of Morris-Shin, it is interesting that the determinacy of equilibrium in both cases hinges on exactly the same ratio $\alpha_z/\sqrt{\alpha_x}$. However, note that, in equilibrium, the information generated by y is equivalent to the information generated by $z = Z(y) = \theta - \sigma_x \varepsilon$. Our endogenous-information economy is thus related to an exogenous-information economy with precision of public information given by $\alpha_z = \alpha_y = \alpha_\varepsilon \alpha_x$. Indeed, substituting this expression into the criterion for multiplicity from proposition, ?? that $\sigma_x/\sigma_z^2 > \sqrt{2\pi}$, yields the criterion for multiplicity in proposition 3, that $\sigma_\varepsilon^2 \sigma_x < 1/\sqrt{2\pi}$. As the precision of private information becomes infinite so does the precision of public information.

Proposition 3 establishes that, for any given level of noise in the agents' private information, multiple equilibria exist if and only if the noise in the macroeconomic indicator is sufficiently small. Intuitively, an increase in σ_ε reduces the public information generated by the observation of y and thus reduces the ability of the market to coordinate on multiple courses of action. Indeed, the equilibrium conditions (12) and (13) imply that, for every y, we have $\theta^*(y) \to 1 - c/b = \widehat{\theta}$ and $x^*(y) \to \widehat{\theta} + \sigma_x \Phi^{-1}(\widehat{\theta})$ as $\sigma_\varepsilon \to \infty$.

Proposition 4 (Limit $\sigma_\varepsilon \to \infty$) *Fix σ_x and let $\sigma_y \to \infty$. In the limit, the regime changes if and only if $\theta \leq \widehat{\theta}$, where $\widehat{\theta} \equiv 1 - c/b \in (\underline{\theta}, \overline{\theta})$.*

The Morris-Shin outcome is obtained as the noise in the observation of aggregate activity becomes arbitrarily large. This is intuitive, for in this case no information is generated by the observation of y and the endogeneity of public information is of no importance.

Consider next the limit of the precision of agents' private information, for given level of noise in y. Proposition (3) establishes that, for given σ_ε, multiple equilibria exist if and only if σ_x is sufficiently small. The interval $(\underline{z}, \overline{z})$ represents the region of multiplicity and a

---

[4]If $A = 0$, then $y = -\infty$ and $x^*(y) = \Phi^{-1}(-\infty) = \underline{\theta}$, in which case all agents attack whenever $\theta \leq \underline{\theta}$ and no agent attacks whenever $\theta > \underline{\theta}$. If instead $A = 1$, then $y = +\infty$ and $x^*(y) = \Phi^{-1}(+\infty) = \overline{\theta}$, in which case all agents attack whenever $\theta \leq \overline{\theta}$ and no agent attacks whenever $\theta > \overline{\theta}$. In the former case, a regime change is triggered if and only if $\theta \leq \underline{\theta}$; in the latter, if and only if $\theta \leq \overline{\theta}$.

reduction in σ_x reduces $\underline{z}$ and increases $\bar{z}$ making larger the multiplicity region. Indeed, as $\sigma_x \to 0$ we can show that any outcome is possible for all $\theta \in (\underline{\theta}, \bar{\theta})$.

Proposition 5 (Limit $\sigma_x \to 0$) *Fix σ_ϵ and let $\sigma_x \to 0$. There exists an equilibrium with the probability of regime change converging to zero for all $\theta \in (\underline{\theta}, \bar{\theta})$, as well as an equilibrium with the probability of regime change converging to one for every $\theta \in (\underline{\theta}, \bar{\theta})$.*

Proof. First, note that $\underline{y} \to -\infty$ and $\bar{y} \to +\infty$ as $\sigma_x \to 0$. Next, note that both $|\sigma_\epsilon^2 \sigma_x - \phi(\underline{y})|$ and $|\sigma_\epsilon^2 \sigma_x - \phi(\bar{y})|$ vanish. Since $\lim_{y \to -\infty} \phi(y)y = \lim_{y \to +\infty} \phi(y)y = 0$, the latter implies $\sigma_x \underline{y} \to 0$ and $\sigma_x \bar{y} \to 0$. Hence, $\underline{z} \to \Phi(-\infty) = \underline{\theta}$ and $\bar{z} \to \Phi(+\infty) = \bar{\theta}$ as $\sigma_x \to 0$. Moreover, for every θ and ε, $\theta - \sigma_x \varepsilon \to \theta$ as $\sigma_x \to 0$. It follows that

$$\Pr \left[\, \theta - \sigma_x \varepsilon \in (\underline{z}, \bar{z}) \mid \theta \in (\underline{\theta}, \bar{\theta}) \, \right] \to 1 \quad \text{as} \quad \sigma_x \to 0.$$

Next, let $\underline{Y}(\theta, \varepsilon) \equiv \min \mathcal{Y}(\theta - \sigma_x \varepsilon)$ and $\bar{Y}(\theta, \varepsilon) \equiv \max \mathcal{Y}(\theta - \sigma_x \varepsilon)$ and consider (θ, ε) such that $\theta - \sigma_x \varepsilon \in (\underline{z}, \bar{z})$. Note that $\underline{Y}(\theta, \varepsilon) < \underline{y} < \bar{y} < \bar{Y}(\theta, \varepsilon)$ and therefore

$$\underline{Y}(\theta, \varepsilon) \to -\infty \quad \text{and} \quad \bar{Y}(\theta, \varepsilon) \to +\infty \quad \text{as} \quad \sigma_x \to 0.$$

From (12), $\theta^*(y)$ is independent of σ_x, $\theta^*(y) \to \Phi(-\infty) = \underline{\theta}$ as $y \to -\infty$, and $\theta^*(y) \to \Phi(+\infty) = \bar{\theta}$ as $y \to +\infty$. It follows that, as long as $\theta \in (\underline{\theta}, \bar{\theta})$,

$$\Pr \left[\, \theta \leq \theta^* \left(\underline{Y}(\theta, \varepsilon) \right) \, \right] \to 0 \quad \text{and} \quad \Pr \left[\, \theta \leq \theta^* \left(\bar{Y}(\theta, \varepsilon) \right) \, \right] \to 1 \quad \text{as} \quad \sigma_x \to 0,$$

which establishes the result. ∎

This result stands in stark contrast to the Morris-Shin limit result in Proposition 2. With exogenous public information a unique equilibrium survives as the noise in private information vanishes. With endogenous public information as modeled here the multiplicity present with common knowledge obtains as the noise in private information vanishes. The reason is once again the endogeneity of public information: As the precision of private information increases, the precision of public information also increases, and indeed at the same rate, so that common knowledge is recovered in the limit.

3.3 Non-Simultaneous Signal

The analysis so far has assumed that agents can condition their decision to attack on a noisy indicator of *contemporaneous* aggregate behavior. We now consider an alternative

that introducing some simple dynamics to break the simultaneity of signals and actions. As a result, the equilibrium concept in this model is entirely game-theoretic.

There are two types of agents, "early" and "late". Early agents move first, on the basis of their private information alone. Late agents move second, on the basis of their private information as well as a noisy public signal about the aggregate activity of early agents. Neither group can observe contemporaneous activity, but late agents can condition their behavior on the activity of early agents.

Let $\mu \in (0,1)$ denote the fraction of early agents, A_1 the aggregate activity of early agents, and A_2 the aggregate activity of late agents. The regime changes if and only if $\mu A_1 + (1-\mu)A_2 \geq \theta$. The signal generated by early agents and observed only by late agents is given by

$$y_1 = \Phi^{-1}(A_1) + \varepsilon, \tag{15}$$

where $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ is independent of θ and ξ.

In a monotone equilibrium, an early agent attacks if and only if $x \leq x_1^*$, for some threshold x_1^*. The aggregate attack of early agents is thus

$$A_1(\theta) = \Phi\left(\sqrt{\alpha_x}\,[\theta - x_1^*]\right). \tag{16}$$

Hence, in equilibrium

$$y_1 = \Phi^{-1}(A_1(\theta)) + \varepsilon = \sqrt{\alpha_x}\,[x_1^* - \theta] + \varepsilon.$$

The observation of y is thus equivalent to the observation of a public signal z, which is defined by

$$z = x_1^* - \frac{1}{\sqrt{\alpha_x}}y = \theta - \sigma_x\varepsilon.$$

The strategy of a late agent is contingent on his private signal x and the public signal y. Since y and z has the same informational content, we can equivalently express the strategy of a late agent as a function of x and z. Hence, in a monotone equilibrium, a late agent attacks if and only if $x \leq x_2^*(z)$, for some threshold function x_2^*. It follows that the aggregate attack of late agents is

$$A_2(\theta, z) = \Phi\left(\sqrt{\alpha_x}\,[\theta - x_2^*(z)]\right). \tag{17}$$

Combining (16) and (17), we obtain the overall size of attack:

$$A(\theta, z) = \mu\Phi\left(\sqrt{\alpha_x}\,[\theta - x_1^*]\right) + (1-\mu)\Phi\left(\sqrt{\alpha_x}\,[\theta - x_2^*(z)]\right).$$

It follows that the regime changes if and only if $\theta \leq \theta^*(z)$, where $\theta^*(z)$ solves $A(\theta^*(z), z) = \theta^*(z)$, or equivalently

$$\mu \Phi \left(\sqrt{\alpha_x} \left[\theta^*(z) - x_1^* \right] \right) + (1 - \mu) \Phi \left(\sqrt{\alpha_x} \left[\theta^*(z) - x_2^*(z) \right] \right) = \theta^*(z). \tag{18}$$

Next, consider the optimal behavior of the agents. Note that the realization of z is known to the late agents but unknown to the early agents. The threshold $x_2^*(z)$ thus solves $b \Pr \left[\theta \leq \theta^*(z) | x_2^*(z), z \right] = c$, or equivalently

$$\Phi \left(\sqrt{\alpha} \left(\delta x_2^*(z) + (1 - \delta) z - \theta^*(z) \right) \right) = \frac{b - c}{b}, \tag{19}$$

where $\delta = \alpha_x / (\alpha_x + \alpha_z)$ and $\alpha = \alpha_x + \alpha_z$. The threshold x_1^*, on the other hand, solves $b \Pr \left[\theta \leq \theta^*(y) | x_1^* \right] = c$, or equivalently

$$\int \Phi \left(\sqrt{\alpha_x} \left[\theta^*(z) - x_1^* \right] \right) \sqrt{\alpha_1} \phi \left(\sqrt{\alpha_1} \left[z - x \right] \right) dz = \frac{b - c}{b}, \tag{20}$$

where we used the fact that, conditional on x, z is distributed normal with precision $\alpha_1 = \alpha_x \alpha_\varepsilon / (1 + \alpha_\varepsilon)$.[5]

Solving (18) for $x_2^*(z)$ gives

$$x_2^*(z) = \theta^*(z) - \sqrt{\alpha_x} \Phi^{-1} \left(\theta^*(z) + \frac{\mu}{1 - \mu} \left[\theta^*(z) - \Phi \left(\sqrt{\alpha_x} \left[\theta^*(z) - x_1^* \right] \right) \right] \right).$$

Substituting the above into (19) and using $\delta = \alpha_x / (\alpha_x + \alpha_z)$ and $\alpha = \alpha_x + \alpha_z$, we obtain:

$$\Gamma(\theta^*(z), z, x_1^*, \mu) = g, \tag{21}$$

where $g = \sqrt{1 + \alpha_z / \alpha_x} \Phi^{-1} (1 - c/b)$ and

$$\Gamma(\theta, z, x_1, \mu) = \frac{\alpha_z}{\sqrt{\alpha_x}} (z - \theta) + \Phi^{-1} \left(\theta + \frac{\mu}{1 - \mu} \left[\theta - \Phi \left(\sqrt{\alpha_x} \left[\theta - x_1 \right] \right) \right] \right).$$

For any $x_1^* \in \mathbb{R}$ and any $z \in \mathbb{R}$, we have that $\Gamma(\theta, z, x_1^*, \mu)$ is continuous in θ, with $\Gamma(\underline{\theta}, z, x_1^*, \mu) = -\infty$ and $\Gamma(\overline{\theta}, z, x_1^*, \mu) = \infty$. Hence, for any given threshold $x_1^* \in \mathbb{R}$, condition (21) determines a function $\theta^* : \mathbb{R} \to [\underline{\theta}, \overline{\theta}]$. On the other hand, for any given function $\theta^* : \mathbb{R} \to [\underline{\theta}, \overline{\theta}]$, condition (20) determines a threshold $x_1^* \in \mathbb{R}$. An equilibrium is any joint solution to (20) and (21).

---

[5]To see this, note that $z = \theta - \sigma_x \varepsilon = x - \xi - \sigma_x \varepsilon$, so that $z | x \sim \mathcal{N} \left(0, \sigma_x^2 + \sigma_x^2 \sigma_\varepsilon^2 \right)$.

Consider now the limit as $\mu \to 0$. Note that, for all $(\theta, z, x_1^*) \in \mathbb{R}^2$, $\mu \to 0$ implies

$$\Gamma(\theta, z, x_1^*, \mu) \to G(\theta, z) = \frac{\alpha_z}{\sqrt{\alpha_x}}(z - \theta) + \Phi^{-1}(\theta).$$

Note that G is independent of x_1^* and is the same as in the Morris-Shin benchmark. Consider now a function $\theta^* : \mathbb{R} \to [\underline{\theta}, \overline{\theta}]$ such that, for all z,

$$G(\theta^*(z), z) = g. \tag{22}$$

As shown earlier, θ^* is unique if and only if $\alpha_z/\sqrt{\alpha_x} \le \sqrt{2\pi}$. If instead $\alpha_z/\sqrt{\alpha_x} > \sqrt{2\pi}$, there are multiple θ^* solving (22). Moreover, for any such θ^*, (20) admits a unique solution $x_1^* \in \mathbb{R}$. We conclude that, for μ small enough, there are multiple solutions to (20) and (21) whenever $\alpha_z/\sqrt{\alpha_x} > \sqrt{2\pi}$. But $\alpha_z = \alpha_\varepsilon \alpha_x$, so that multiple equilibria again emerge as long as $\alpha_\varepsilon \sqrt{\alpha_x} > \sqrt{2\pi}$.

Moreover, the equilibria of this economy approximate the equilibria of our benchmark model in the following sense. Let $\mathcal{E}_{(\mu)}$ denote the "dynamic" economy of this section and $\mathcal{E}$ the "static" economy of the previous section. Let x^* and θ^* denote the equilibrium thresholds for $\mathcal{E}$, $\mathcal{Y}$ the correspondence defined in (10), and $\widetilde{Y}$ a function selected from this correspondence. For any $\omega > 0$, we can find μ small enough such that $\left| \theta^*_{(\mu)}(z) - \theta^*(\widetilde{Y}(z)) \right| < \omega$ and $\left| x^*_{2(\mu)}(z) - x^*(\widetilde{Y}(z)) \right| < \omega$ for all z, where $x^*_{2(\mu)}$ and $\theta^*_{(\mu)}$ are the thresholds associated with an equilibrium of $\mathcal{E}_{(\mu)}$. To see this, note that θ^* and $\widetilde{Y}$ are part of an equilibrium for $\mathcal{E}$ if and only if the composite $\theta^* \circ \widetilde{Y}$ is a solution to (22).

Finally, let us introduce a random variable y_2 defined by

$$y_2 = \Phi^{-1}(A_2) + \varepsilon,$$

where ε is the same realization as the one in (15). y_2 is a noisy indicator about the activity of late agents. If y_2 is unobservable, late agents continue to attack if and only if $x \le x_2^*(z)$. Hence, in equilibrium, $\Phi^{-1}(A_2) + \varepsilon = \sqrt{\alpha_x}[x_2^*(z) - z]$ and y_2 is a function of z alone:

$$y_2(z) = \sqrt{\alpha_x}(x_2^*(z) - z).$$

Since y_2 conveys no more information than z and thus no more information than y_1, nothing changes if we allow late agents to condition their behavior on y_2 in addition to y_1. That is, the equilibria of the economy where late agents observe both y_1 and y_2 coincide with the equilibria of the economy where late agents observe only y_1.

Now consider again the limit as $\mu \to 0$. Since $x^*_{2(\mu)}(z) \to x^*(\widetilde{Y}(z))$, we have $y_{2(\mu)}(z) \to$

18

$\sqrt{\alpha_x} \left[x^*(\widetilde{Y}(z)) - z \right]$. By definition of $\widetilde{Y}(z)$, we have $\widetilde{Y}(z) = \sqrt{\alpha_x} \left[x^*(\widetilde{Y}(z)) - z \right]$. Hence, $y_{2(\mu)}(z) \to \widetilde{Y}(z)$. That is, in the limit as $\mu \to 0$, the random variable $y_{2(\mu)}$ that is part of an equilibrium in economy $\mathcal{E}_{(\mu)}$ solves the fixed-point condition $y = \Phi^{-1}(A(\theta; y)) + \varepsilon$ for economy $\mathcal{E}$. Conversely, any $\widetilde{Y}(z)$ that is part of an equilibrium for $\mathcal{E}$ can be approximated by a random variable $y_{2(\mu)}$ of economy $\mathcal{E}_{(\mu)}$. This indeed provides a justification for the equilibrium selection we made in Section 3.1. Any equilibrium of the "static" economy in which the signal y is not a function of z alone, if it exists, can not be approximated by an equilibrium of a "dynamic" economy.

4   Financial Prices

The analysis so far has assumed that agents can condition their decision to attack on a noisy indicator of aggregate behavior. Instead, we now allow agents to observe a *financial price* that is determined earlier in a competitively asset market.

We modify the environment as follows. There are two stages and we refer to stage 1 as the 'asset market'. All individuals begin with the same endowment of wealth w and an exogenous private signal $x_i = \theta + \xi_i$, where the noise ξ_i is as before.

In the first stage, agents trade a financial asset that has a dividend that depends on the underlying fundamentals, either directly or indirectly. In the second stage, agents decide whether to attack the regime or not as in the basic model. The return of the asset and the regime outcome are realized at the end of the second stage. Here, in deciding whether to attack agents can no longer condition their choice on a direct signal of the aggregate attack as in Section 3. However, in addition to their private information, they can use information revealed by the equilibrium price from the first stage.

This framework opens up new modeling choices regarding the specification of the financial asset's payoff and the preferences over risky payoffs. Following Grossman-Stiglitz, we guide our choice with an eye towards tractability. The four specifications we solve below are designed so that they preserve normality of the information structure and are solvable in closed-form.

We first introduce some general notation and state the equilibrium conditions. We denote by p the price of the asset or more generally some measure of the terms of trade. Let f the dividend paid by the asset is represented by f and k_i be the investment agent i makes in this asset. For all cases we consider we can express the indirect utility the agent enjoys from his portfolio choice by a function $V(k, f, p)$ so that the total payoff is

$$u_i = V(k_i, f, p) + U(a_i, R),$$

where the utility from attacking U is just as in Section 2.

Let aggregate demand for the asset be $K = \int k_i di$. We assume there is a shock ε to the exogenous net supply of the asset. One interpretation of the net supply shock is that it results from a shock to the demand of other 'noisy' traders. This shock is not observed by individuals and we assume that it is $\mathcal{N}(0, \sigma_\varepsilon^2)$ and independent of both the fundamentals and the private noise.

Market clearing requires $K = \varepsilon$ which determines an equilibrium price function $P(\theta, \varepsilon)$. As in Grossman and Stiglitz (1976, 1980), the role of the shock ε is to introduce noise in the information revealed by financial prices about fundamentals. Since the price function is a function of both θ and ε the observation of p does not reveal θ perfectly, leading to a signal-extraction problem.

A *rational-expectations equilibrium* is a price function, $p = P(\theta, \varepsilon)$, individual strategies for investment and attacking, $k(x^i, p)$ and $a(x^i, p)$, and aggregate investment and attack functions, $K(\theta, p)$ and $A(\theta, \varepsilon)$, such that in the first stage:

$$k(x, p) = \arg \max_{k \in \mathbb{R}} \mathbb{E}\left[V(k, f, p) \mid x, p \right] \tag{23}$$

$$K(\theta, p) = \int_x k(x, p) d\Phi\left(\frac{x - \theta}{\sigma_x} \right) \tag{24}$$

$$K(\theta, P(\theta, \varepsilon)) = \varepsilon \tag{25}$$

and in the second stage

$$a(x, p) = \arg \max_{a \in [0,1]} \mathbb{E}\left[U(a, R) \mid x, p \right], \tag{26}$$

$$A(\theta, p) = \int_x a(x, p) d\Phi\left(\frac{x - \theta}{\sigma_x} \right), \tag{27}$$

where $R = 1$ if $A(\theta, p) \geq \theta$ and $R = 0$ otherwise.

The information an agent has consists of the privately observed signal x and the publicly observed price p. In this sense, the price p takes the place that the signal y had in the observable action model of Section . Condition (25) imposes market clearing in the asset market. Condition (23) and (27) requires that an agent's investment take into account the information contained in their private information and prices.

The four specifications we consider below generate the following information structure.

There is a strictly monotone function $Z(y)$ and a random variable v that is $\mathcal{N}(0, Q(\alpha_x, \alpha_\varepsilon)^{-1})$ and independent of θ and ξ such that $Z(P(\theta, \varepsilon)) = \theta + v$ for every realization of θ and ε. Thus, the observation of an equilibrium price realization p is informationally equivalent to the observation of a public signal $z = Z(p)$ on θ with normal error and precision $\alpha_p = Q(\alpha_x, \alpha_\varepsilon)$.

The agent's posterior conditional on his private information x and the observed price p is

$$\theta \mid x, p \sim \mathcal{N}\left(\delta x + (1 - \delta)z , \ \alpha^{-1} \right),$$

where $\delta = \alpha_x/(\alpha_x + \alpha_p)$ and $\alpha = \alpha_x + \alpha_p$. Like in the Morris-Shin benchmark, the determinacy of equilibrium turns out to depend on the ratio

$$\frac{\alpha_p}{\sqrt{\alpha_x}},$$

that is, the ratio of the precision of the public information generated by the price to the square root of the precision of the exogenous private information. If $\alpha_p/\sqrt{\alpha_x} < \sqrt{2\pi}$, then the equilibrium is unique. If instead $\alpha_p/\sqrt{\alpha_x} > \sqrt{2\pi}$, then there are multiple equilibria. Unlike Morris-Shin, however, the precision of public information α_p, and therefore the ratio $\alpha_p/\sqrt{\alpha_x}$, are endogenous and affected by σ_x.

In what follows, we consider four alternative specifications of the asset market (stage 1). In each case, we solve for the equilibrium price function and the associated mappings Z and Q. We then examine the critical ratio $\alpha_p/\sqrt{\alpha_x}$ to study the determinacy of equilibrium as a function of the exogenous information structure, $(\alpha_x, \alpha_\varepsilon)$, or equivalently $(\sigma_x, \sigma_\varepsilon)$.

4.1 Risk Aversion – Fundamental Dividend

We start with an example that maps directly to the CARA-normal framework introduced by Grossman and Stiglitz (1976, 1980) [see also Hellwig (1980)]. The agent can invest his wealth either in a risky asset or a risk-less asset. We normalize the gross return of the risk-less asset: it costs the agent 1 in the first stage and delivers 1 in the second stage. The risky asset costs the agent p in the first stage and delivers $f(\theta) = \theta$ in the second. Here the return of the asset depends directly on the exogenous fundamental.

The agent enjoys utility only from second-stage consumption and his preferences over second-stage consumption exhibit constant absolute risk aversion (CARA). The indirect utility from his portfolio choice is thus given by

$$V(k, f, p) = u\left(w - pk + fk\right), \quad u(c) = -\exp(-2\gamma c)/2\gamma, \tag{28}$$

where k is the amount invested in the risky asset and $w - pk$ is invested in the risk-less asset, $c = w - pk + fk$ is second-period consumption.

Setting $\frac{\partial}{\partial k}\mathbb{E}[\, V(k, f, p) \mid x, p] = 0$ we obtain,

$$k(x, p) = \frac{\mathbb{E}[\, \theta \mid x, p\,] - p}{\gamma \text{Var}[\, \theta \mid x, p\,]}.$$

We then guess and verify that

$$\mathbb{E}[\, \theta \mid x, p\,] = \delta x + (1 - \delta)p \quad \text{and} \quad \text{Var}[\, \theta \mid x, p\,] = \alpha^{-1},$$

for some $\delta \in (0, 1)$ and $\alpha > 0$, in which case the optimal demand reduces to

$$k(x, p) = \frac{\delta \alpha}{\gamma}(x - p).$$

It follows that the aggregate demand for the asset is

$$K(\theta, p) = \frac{\delta \alpha}{\gamma}(\theta - p).$$

In equilibrium, $K \doteq \int k_i di = \varepsilon$, and therefore the equilibrium price satisfies

$$p = P(\theta, \varepsilon) = \theta - \frac{\gamma}{\delta \alpha}\varepsilon. \tag{29}$$

By implication, the observation of p is equivalent to the observation of a public signal about θ with precision $(\delta \alpha / \gamma)^2 \alpha_\varepsilon$. That is, in this case we have $Z(p) = p$, $v = -(\delta \alpha / \gamma)\varepsilon$, and $\alpha_p = (\delta \alpha / \gamma)^2 \alpha_\varepsilon$.

We now determine δ and α. Note that x and $Z(p) = p$ are independent signals of θ with precision α_x and α_p, respectively. It follows that $\mathbb{E}[\, \theta \mid x, p\,] = \delta x + (1 - \delta)p$, where

$$\delta = \frac{\alpha_x}{\alpha_x + \alpha_p} = \frac{\alpha_x}{\alpha}$$

$$\alpha = \alpha_x + \alpha_p = \alpha_x + (\delta \alpha / \gamma)^2 \alpha_\varepsilon$$

Solving the above gives $\alpha = \alpha_x(1 + \alpha_x \alpha_\varepsilon / \gamma^2)$, $\delta = 1/(1 + \alpha_x \alpha_\varepsilon / \gamma^2)$, and

$$\alpha_p = Q(\alpha_\varepsilon, \alpha_x) = \frac{\alpha_\varepsilon \alpha_x^2}{\gamma^2}. \tag{30}$$

Recall that in Section 3 we found that the precision of endogenous information increased proportionally with the precision of private information. Here the precision of public information

increases *more* than proportionally with the precision of private information. This will only serve to reinforce our conclusions regarding the comparative static and limit exercises for σ_x.

To verify this, consider stage 2. A monotone (continuation) equilibrium is characterized by thresholds $x^*(p)$ and $\theta^*(p)$ such that an agent attacks if and only if $x \leq x^*(p)$ and the regime changes if and only if $\theta \leq \theta^*(p)$. The threshold $x^*(p)$ solves $b \Pr[\theta \leq \theta^*(p)|x,p] = c$, or equivalently

$$\Phi\left(\sqrt{\alpha_x + \alpha_p}\left(\frac{\alpha_x}{\alpha_x + \alpha_p}x^*(p) + \frac{\alpha_p}{\alpha_x + \alpha_p}Z(p) - \theta^*(p)\right)\right) = \frac{b-c}{b}. \tag{31}$$

The threshold $\theta^*(p)$, on the other hand, solves $A(\theta^*(p), p) = \theta^*(p)$, or equivalently

$$x^*(p) = \theta^*(p) + \frac{1}{\sqrt{\alpha_x}}\Phi^{-1}(\theta^*(p)). \tag{32}$$

Combining the above two conditions and using $Z(p) = p$, we have that $\theta^*(p)$ can be sustained in equilibrium if and only if it solves

$$\frac{\alpha_p}{\sqrt{\alpha_x}}(p - \theta^*(p)) + \Phi^{-1}(\theta^*(p)) = \sqrt{\frac{\alpha_x + \alpha_p}{\alpha_x}}\Phi^{-1}(1 - c/b). \tag{33}$$

Similar arguments as those used for Proposition 1 imply that there are multiple $\theta^*(p)$ if and only if $\alpha_p/\sqrt{\alpha_x} > \sqrt{2\pi}$, where α_p is given by (30). On the other hand, the price function is given by (29) and is uniquely determined.

Proposition 6 *Suppose V is given by (28) and $f = 0$. The equilibrium price function $P(\theta, \varepsilon)$ is always uniquely determined. There are multiple equilibrium thresholds $x^*(p)$ and $\theta^*(p)$ if and only if $\sigma_\varepsilon^2 \sigma_x^3 < \gamma^2 (2\pi)^{-1/2}$.*

As in the benchmark model, multiple equilibria survive as long as either noise is small enough. Indeed, the common-knowledge outcomes are once again obtained in the limit as $\sigma_x \to 0$ for any given σ_ε.

4.2 Risk Aversion – Speculative Dividend

We now modify the previous example by letting $f = f(A) = -\Phi^{-1}(A)$. That is, the dividend of the asset is now a function of the equilibrium size of the attack realized in the second stage. We earlier showed that, in equilibrium, $A = \Phi\left(\sqrt{\alpha_x}[x^*(p) - \theta]\right)$. It follows that

$f = \sqrt{\alpha_x}[\theta - x^*(p)]$ and therefore

$$k = \frac{\sqrt{\alpha_x}\mathbb{E}[\,\theta \mid x, p\,] - \sqrt{\alpha_x}x^*(p) - p}{\gamma\alpha_x\text{Var}[\,\theta \mid x, p\,]}.$$

Let

$$\widetilde{p} = \frac{1}{\sqrt{\alpha_x}}p + x^*(p), \tag{34}$$

and note that, for every p, the above de□nes a unique $\widetilde{p}$. We can then rewrite the optimal demand as

$$k = \frac{\mathbb{E}[\,\theta \mid x, p\,] - \widetilde{p}}{\widetilde{\gamma}\text{Var}[\,\theta \mid x, p\,]},$$

where $\widetilde{\gamma} = \gamma\sqrt{\alpha_x}$. The rest is then as in the previous example, provided we replace γ with $\widetilde{\gamma}$. In particular, we have

$$\widetilde{p} = \theta - \frac{\widetilde{\gamma}}{\delta\alpha}\varepsilon, \tag{35}$$

so that $Z(p) = \widetilde{p}$, $v = -(\delta\alpha/\widetilde{\gamma})\varepsilon$, and $\alpha_p = \alpha_\varepsilon\alpha_x^2/\widetilde{\gamma}^2$. Using $\widetilde{\gamma} = \gamma\sqrt{\alpha_x}$, we conclude that the precision of the information revealed by the price is now given by

$$\alpha_p = Q(\alpha_\varepsilon, \alpha_x) = \frac{\alpha_\varepsilon\alpha_x^3}{\gamma^2}. \tag{36}$$

Once again, the precision of public information increases *more* than proportionally with the precision of private information, which only reinforces our results.

Indeed, consider stage 2. As in the previous example, the thresholds $x^*(p)$ and $\theta^*(p)$ solve (31) and (32). The difference is that now the endogenous signal is given by $Z(p) = \widetilde{p} = \frac{1}{\sqrt{\alpha_x}}p + x^*(p)$. Hence, (??) is now replaced by

$$\theta^*(p) = \Phi\left(\frac{\sqrt{\alpha_x}}{\sqrt{\alpha_x + \alpha_p}}\Phi^{-1}(1 - c/b) - \frac{\alpha_p}{\alpha_x + \alpha_p}p\right), \tag{37}$$

where α_p is given by (36). It follows that the threshold $\theta^*(p)$ and $x^*(p)$ are uniquely determined. What may be indeterminate now is the price function.

Using (34), (35), and (37), we have that p must solve

$$F(p) \equiv \Phi\left(-\frac{\alpha_p}{\alpha_x + \alpha_p}p + \Lambda\right) + \frac{1}{\sqrt{\alpha_x}}\left[\frac{\alpha_x}{\alpha_x + \alpha_p}p + \Lambda\right] = z \tag{38}$$

where $\Lambda = \Phi^{-1}\left(\frac{b-c}{b}\right)\sqrt{\alpha_x}/\sqrt{\alpha_x + \alpha_p}$ and $z = \theta - (\delta\alpha/\widetilde{\gamma})\varepsilon$. This equation is analogous to equation (14) in the benchmark model. Note that $F(p)$ is continuous in p, and $F(p) \to -\infty$ as $p \to -\infty$, and $F(p) \to +\infty$ as $p \to +\infty$, which implies that a solution always exists.

Moreover,

$$F'(p) = \frac{\alpha_p}{\alpha_x + \alpha_p} \left(\frac{\sqrt{\alpha_x}}{\alpha_p} - \phi \left(-\frac{\alpha_p}{\alpha_x + \alpha_p} p + \Lambda \right) \right),$$

so the solution is unique for all z if $\alpha_p/\sqrt{\alpha_x} < \sqrt{2\pi}$. If instead $\alpha_p/\sqrt{\alpha_x} > \sqrt{2\pi}$, there are thresholds $\underline{z}$ and $\overline{z}$ such that there exist multiple equilibrium prices whenever $z \in (\underline{z}, \overline{z})$.

Proposition 7 *Suppose V is given by (28) and $f = -\Phi^{-1}(A)$. The equilibrium thresholds $x^*(p)$ and $\theta^*(p)$ are always uniquely determined. There are multiple equilibrium price functions $P(\theta, \varepsilon)$ if and only if $\sigma_\varepsilon^2 \sigma_x^3 < \gamma^2/\sqrt{2\pi}$.*

The results here are reminiscent of the ones in the benchmark model. In equilibrium, the price plays the role of an anticipatory signal of the size of the attack. Indeed, as we found there, the strategies of the agents are uniquely determined, although the equilibrium price function may not be. In contrast, in the previous example the price played the role of a signal for the exogenous fundamental θ. Here indeterminacy arises for individual strategies and not for the price function.

In both cases endogeneity of public information implies that its precision is increasing with the precision of private information. In both cases this implies that multiplicity survives with small noise and the common-knowledge outcomes obtain in the limit as $\sigma_x \to 0$ for any given σ_ε.

4.3 Risk Neutrality – Fundamental Dividend

We modify the asset market and the preferences as follows. There is no risk-less bond. One unit of the asset costs 1 in the first period and pays $f - p$ in the second period. The indirect utility from the portfolio choice is thus given by

$$V(k, f, p) = u_1(w - k) + u_2((f - p)k) \tag{39}$$

where k is the amount invested, u_1 is the utility from first-period consumption, u_2 is the utility from second-period consumption, and U is the payoff from attacking.

Consider an agent who receives a private signal x and observes a price p. His optimal investment k solves

$$u_1'(w - k) = \mathbb{E}[(f - p)u_2'((f - p)k) \mid x, p]. \tag{40}$$

We assume that $u_1(c)$ is quadratic and $u_2(c)$ is linear, in which case (40) reduces to a simple linear relation, $k_i = \kappa \{\mathbb{E}[f|x, p] - p\} + \lambda$, for some constants $\kappa > 0$, $\lambda \in \mathbb{R}$. With out any

loss of generality, we normalize $\lambda = 0$. Finally, we let $f = f(\theta) = \theta$. That is, the return of the asset depends only on the exogenous fundamental.

The analysis here is similar to that in the first example. The optimal individual demand for the asset is

$$k = \kappa \left\{ \mathbb{E}\left[f \mid x, p \right] - p \right\} = \kappa \left\{ \mathbb{E}\left[\theta \mid x, p \right] - p \right\}.$$

We conjecture

$$\mathbb{E}\left[\theta \mid x, p \right] = \delta x + (1 - \delta)p$$

for some $\delta \in (0, 1)$ to be determined. It follows that $k = k(x, p) = \kappa\delta(x - p)$ and therefore $K(\theta, p) = \kappa\delta\left(\theta - p\right)$. In equilibrium, $K = \varepsilon$. Hence, the equilibrium price is

$$p = P(\theta, \varepsilon) = \theta - \frac{1}{\kappa\delta}\varepsilon.$$

By implication, p is a public signal about θ with precision $\kappa^2\delta^2\alpha_\varepsilon$. That is, in this example $Z(p) = p$ and $v = -\frac{1}{\kappa\delta}\varepsilon$. It remains to pin down δ and the function Q.

Note that α_p is bounded above by $\kappa^2\alpha_\varepsilon$ and therefore we immediately have that uniqueness is ensured for α_x high enough. To complete the analysis, note that

$$\delta = \frac{\alpha_x}{\alpha_x + \alpha_p} = \frac{\alpha_x}{\alpha_x + \alpha_\varepsilon\delta^2\kappa^2}.$$

The above uniquely determines $\delta \in (0, 1)$ as an increasing function of α_u and a decreasing function of α_ε. To see this, let $\alpha = \alpha_x/\left(\alpha_\varepsilon\kappa^2\right)$ and rewrite the above as $\alpha = \delta^3/(1 - \delta)$. Obviously, this gives a monotonic relation between α and δ, with $\delta \to 0$ as $\alpha \to 0$ and $\delta \to 1$ as $\alpha \to \infty$. Using these results, we find

$$\frac{\alpha_p}{\sqrt{\alpha_x}} = \frac{\kappa^2\delta^2\alpha_\varepsilon}{\sqrt{\alpha_x}} = \left(\kappa\sqrt{\alpha_\varepsilon}\right)\frac{\delta^2}{\sqrt{\alpha_x}}$$
$$= \left(\kappa\sqrt{\alpha_\varepsilon}\right)\sqrt{\delta\left(1 - \delta\right)}.$$

The fact that $\delta\left(1 - \delta\right) \to 0$ as either $\alpha_x \to 0$ or $\alpha_x \to \infty$ then implies that, given α_ε, we have that $\alpha_p/\sqrt{\alpha_x} < \sqrt{2\pi}$ and therefore the equilibrium in unique if and only if α_x is either sufficiently small or sufficiently high. On the other hand, for given α_x, we have $\delta\left(1 - \delta\right) \le 1/4$ necessarily and therefore $\alpha_\varepsilon < 8\pi/\kappa^2$ is sufficient for uniqueness, whereas α_ε sufficiently high is sufficient for multiplicity. We conclude:

Proposition 8 *Suppose V is given by (39) and $f = \theta$. The equilibrium price function P is always uniquely determined. There are multiple equilibrium thresholds $x^*(p)$ and $\theta^*(p)$ if and only if σ_x is either sufficiently small or sufficiently high relative to σ_ε.*

Thus, the Morris-Shin limit result is preserved in this example. Like in all previous cases we have examined, the precision of public information increases with the precision of private information. Unlike the previous cases, however, this effect is not strong enough to restore multiplicity when σ_x is sufficiently small.

4.4 Risk Neutrality − Speculative Dividend

We modify the previous example by letting the return of the asset be $f = -\Phi^{-1}(A)$, where A is the aggregate size of the attack occurring in the second stage. The analysis is now similar to the second example. In particular, we will show that the strategies in stage 2 are unique but the price function in stage 1 can be indeterminate.

Let $x^*(p)$ denote the threshold agents use in stage 2 in deciding whether to attack. In equilibrium,

$$A = A(\theta, p) = \Phi^{-1}\left(\frac{x^*(p) - \theta}{\sigma_x}\right),$$

so that the asset return is $f = \sqrt{\alpha_x}[\theta - x^*(p)]$. The demand for the asset is thus

$$k = \kappa\left\{\mathbb{E}\left[\,f \mid x, p\,\right] - p\right\} = \kappa\left\{\sqrt{\alpha_x}\mathbb{E}\left[\,\theta \mid x, p\,\right] - p - \sqrt{\alpha_x}x^*(p)\right\}.$$

Let

$$\widetilde{p} = \frac{1}{\sqrt{\alpha_x}}p + x^*(p)$$

and note that, for every p, the above de□nes a unique $\widetilde{p}$. We can thus write the demand as

$$k = \widetilde{\kappa}\left\{\mathbb{E}\left[\,\theta \mid x, p\,\right] - \widetilde{p}\right\}$$

where $\widetilde{\kappa} = \kappa\sqrt{\alpha_x}$. We now conjecture

$$\mathbb{E}\left[\,\theta \mid x, p\,\right] = \delta x + (1 - \delta)\widetilde{p}.$$

It follows that $K = \widetilde{\kappa}\delta(\theta - \widetilde{p})$ and therefore

$$\widetilde{p} = \theta - \frac{1}{\widetilde{\kappa}\delta}\varepsilon.$$

Hence, the observation of p is equivalent to the observation of $\widetilde{p}$, which is a public signal for θ with precision $\alpha_p = \widetilde{\kappa}^2\delta^2\alpha_\varepsilon$. It follows that

$$\mathbb{E}\left[\,\theta \mid x, p\,\right] = \mathbb{E}\left[\,\theta \mid x, \widetilde{p}\,\right] = \delta x + (1 - \delta)\widetilde{p},$$

where

$$\delta = \frac{\alpha_x}{\alpha_x + \alpha_p} = \frac{\alpha_x}{\alpha_x + \alpha_\varepsilon \delta^2 \widetilde{\kappa}^2}.$$

This is the same as in the previous example, with $\widetilde{\kappa}$ replacing κ. Using $\widetilde{\kappa} = \kappa\sqrt{\alpha_x}$, we infer

$$\delta = \frac{1}{1 + \alpha_\varepsilon \delta^2 \kappa^2},$$

so that δ is decreasing in α_ε but independent of α_x. This means that α_p is proportional to α_x, like in the benchmark model. Indeed, the critical ratio is now given by

$$\frac{\alpha_p}{\sqrt{\alpha_x}} = \frac{\widetilde{\kappa}^2 \delta^2 \alpha_\varepsilon}{\sqrt{\alpha_x}} = \left(\kappa^2 \delta^2 \alpha_\varepsilon\right)\sqrt{\alpha_x},$$

and is increasing in both α_ε and α_x. The rest of the analysis is similar to the second example. We conclude:

Proposition 9 *Suppose V is given by (39) and $f = -\Phi^{-1}(A)$. The equilibrium thresholds $x^*(p)$ and $\theta^*(p)$ are always uniquely determined. There are multiple equilibrium price functions $P(\theta, \varepsilon)$ if and only if σ_x and/or σ_ε are sufficiently small.*

Hence, the common-knowledge outcomes are once again obtained as $\sigma_x \to 0$.

5   Final Remarks

Building on Morris and Shin (1998) this paper introduced instruments that endogenized the sources of public information in models where coordination is important. We modeled public information by either: (i) a noisy signal of aggregate activity; or (ii) a financial asset's price that reveals information in equilibrium. An important feature of the equilibrium in all cases is that the precision of public information is endogenous and rises with the precision of private information.

We showed that in all but one of the six models considered this effect is strong enough to reverse the limiting uniqueness result obtained with exogenous public information. Thus, typically, with endogenous public information multiplicity is *ensured* when individuals observe fundamentals with small enough idiosyncratic noise. Conversely, uniqueness is ensured if idiosyncratic noise is large enough.

We view the main theme in Morris-Shin as emphasizing the importance of the details of the information structures for the multiplicity or uniqueness of equilibria. This paper contributes to this same theme by studying the importance of endogeneous information aggregation.

References

[1] Angeletos, George-Marios, Christian Hellwig, and Alessandro Pavan (2003) "Coordination and Policy Traps," MIT/UCLA/Northwestern working paper.

[2] Angeletos, George-Marios, Christian Hellwig, and Alessandro Pavan (2004) "On the Dynamics of Information, Coordination, and Regime Change," MIT/UCLA/Northwestern working paper.

[3] Atkeson, Andrew (2000), "Discussion on Morris and Shin," *NBER Macro Annual.*

[4] Calvo, G. (1988), "Servicing the Public Debt: the Role of Expectations." *American Economic Review,* 78(4), 647-661.

[5] Carlsson, Hans, and Eric van Damme (1993), "Global Games and Equilibrium Selection," *Econometrica* 61, 5, 989-1018.

[6] Cole, Harold, and Timothy Kehoe (1996), "Self-fulílling Debt Crises," Federal Reserve Bank of Minneapolis staff report 211.

[7] Dasgupta, Amil (2002), "Coordination, Learning and Delay," LSE working paper.

[8] Diamond, Douglas, and Philip Dybvig (1983), "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy* 91, 401-19.

[9] Green, Jerry "Information, Efficiency and Equilibrium," Harvard Institute of Economic Research Discussion Paper No. 284, Harvard University.

[10] Grossman, Sanford (1977), "The Existence of Futures Markets, Noisy Rational Expectations and Informational Externalities," *Review of Economic Studies* 44, 431-449.

[11] Grossman, Sanford (1981) "An Introduction to the Theory of Rational Expectations under Asymmetric Information," Review of Economic Studies, 48, 541-559.

[12] Grossman, Sanford, and Joseph Stiglitz (1980), "On the Impossibility of Informationally Efficient Markets," *American Economic Review* 70, 393-408.

[13] Grossman, Sanford, and Joseph Stiglitz (1976), "Information and Competitive Price Systems," *American Economic Review* 66, 246-253.

[14] Hellwig, Martin (1980) "On the Aggregation of Information in Competitive Markets" *Journal of Economic Theory,* 22, 477-498.

[15] Hellwig, Christian (2002), "Public Information, Private Information, and the Multiplicity of Equilibria in Coordination Games," *Journal of Economic Theory*, 107, 191-222.

[16] Morris, Stephen, and Hyun Song Shin (1998), "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks", *American Economic Review*, 88, 3, 587-597.

[17] Morris, Stephen, and Hyun Song Shin (2000), "Rethinking Multiple Equilibria in Macroeconomics," *NBER Macro Annual*.

[18] Morris, Stephen, and Hyun Song Shin (2001), "Global Games - Theory and Applications," in *Advances in Economics and Econometrics*, 8th World Congress of the Econometric Society (M. Dewatripont, L. Hansen, and S. Turnovsky, eds.), Cambridge University Press, Cambridge, UK.

[19] Morris, Stephen, and Hyun Song Shin (2003), "Coordination Risk and the Price of Debt," forthcoming in *European Economic Review*.

[20] Morris, Stephen, and Hyun Song Shin (2004), "Liquidity Black Holes," forthcoming in *Review of Finance*.

[21] Obstfeld, Maurice (1986), "Rational and Self-Fulfilling Balance-of-Payments Crises", *American Economic Review* 76, 1, 72-81.

[22] Obstfeld, Maurice (1996), "Models of Currency Crises with Self-Fulfilling Features," *European Economic Review* 40, 3-5, 1037-47.

[23] Velasco, Andres (1996), "Fixed Exchange Rates: Credibility, Flexibility and Multiplicity," *European Economic Review* 40, 3-5, 1023-36.

Date Due

Lib-26-67